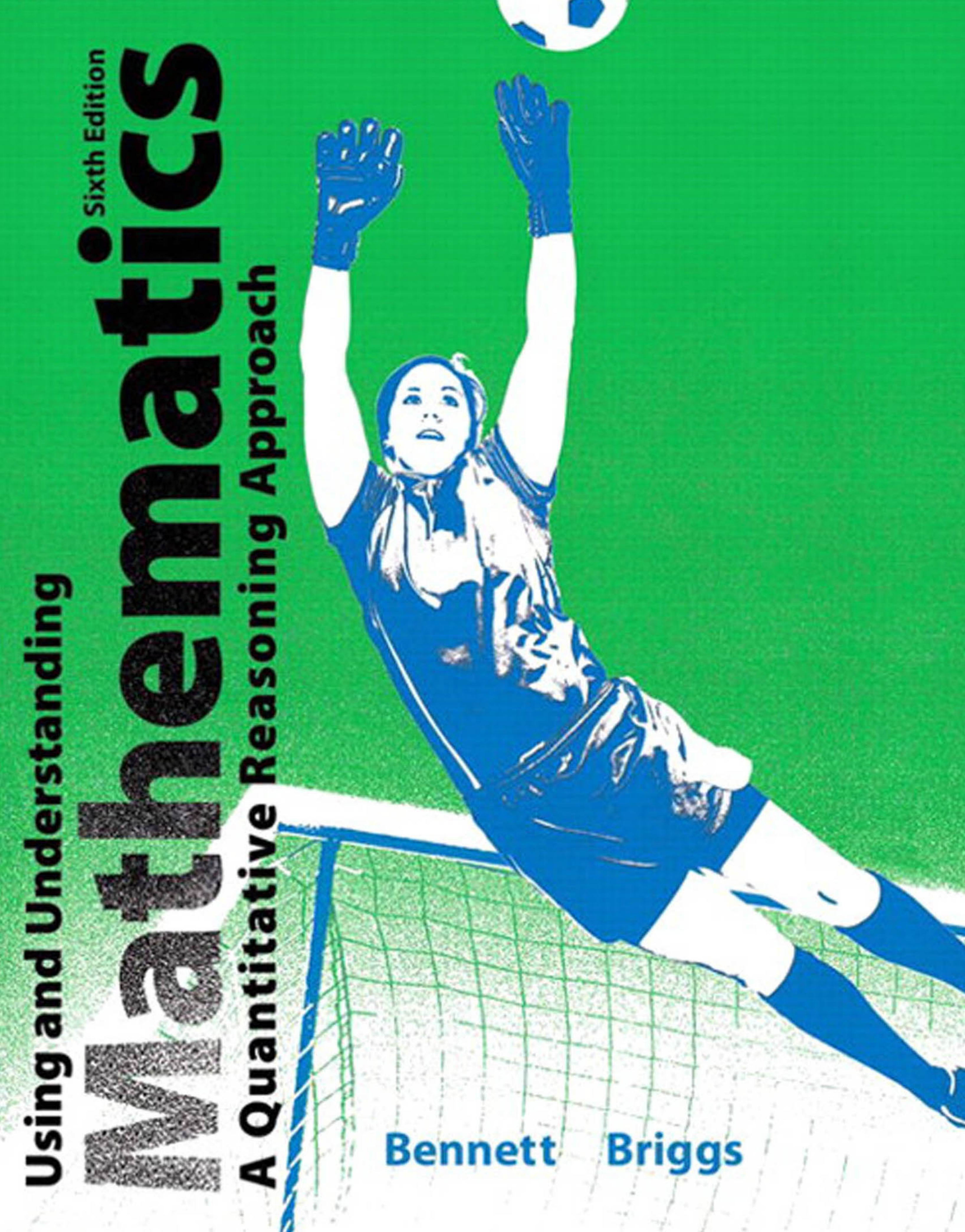
Using and Understanding
Mathematics
Sixth Edition
A Quantitative Reasoning Approach
Bennett Briggs

6TH EDITION

Using & Understanding MATHEMATICS

A Quantitative Reasoning Approach

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This book is dedicated to everyone who wants a better understanding of our world, and especially to those who have struggled with mathematics in the past. We hope this book will help you achieve your goals.

And it is dedicated to those who make our own lives brighter, especially Lisa, Julie, Katie, Grant, and Brooke.

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CONTENTS

PREFACE

"Human history becomes more and more a race between education and catastrophe."

—H. G. Wells, *The Outline of History*, 1920

To the Student

There is no escaping the importance of mathematics in the modern world. However, for most people, the importance of mathematics lies not in its abstract ideas, but in its application to personal and social issues. This book is designed with such practical considerations in mind. In particular, we've designed this book with three specific purposes:

- To prepare you for the mathematics you will encounter in other **college** courses, particularly core courses in social and natural sciences.
- To develop your ability to reason with quantitative information in a way that will help you achieve success in your **career**.
- To provide you with the critical thinking and quantitative reasoning skills needed to understand major issues in **life**.

We hope this book will be useful to everyone, but it is designed primarily for those who are *not* planning to major in a field that requires advanced mathematical skills. In particular, if you've ever felt any fear or anxiety about mathematics, we've written this book with you in mind. Through this book, you will discover that mathematics is much more important and relevant to your life than you had guessed and not as difficult as previously imagined.

Whatever your interests—social sciences, environmental issues, politics, business and economics, art and music, or any of many other topics—you will find many relevant and up-to-date examples in this book. But the most important idea to take away from this book is that mathematics can help you understand a variety of topics and issues, making you a more aware and better educated citizen. Once you have completed your study of this book, you should be prepared to understand most quantitative issues that you will encounter.

To the Instructor

Whether you've taught this course many times or are teaching it for the first time, you are undoubtedly aware that mathematics courses for nonmajors present challenges that differ from those presented by more traditional courses. First and foremost, there isn't even a clear consensus on what exactly should be taught in these courses. While there's little debate about what mathematical content is necessary for science, technology, engineering, and mathematics (STEM) students—for example, these students all need to learn algebra and calculus—there's great debate about what we should teach non-STEM students, especially the large majority who will *not* make use of formal mathematics in their careers or daily lives.

As a result of this debate, core mathematics courses for non-STEM students fall into a broad and diverse range. Some schools require these students to take a traditional, calculus-track course, such as college algebra. Others have instituted courses that teach students about the ways in which contemporary mathematics contributes to society, focusing on mathematical ideas that students are unlikely to encounter elsewhere. These courses have their merits, and they can certainly be made interesting and relevant, but we believe there are better options because of the following important fact: The vast majority (typically 95%) of non-STEM students will *never* take another college mathematics course after completing their core requirements.

Given this fact, we believe it is essential to teach these students the mathematical ideas that they will *need* for their remaining college course work, their careers, and their daily lives. In other words, while there are many topics that might be new and interesting, we must emphasize those topics that are truly important to the future success of these students. The focus of this approach is less on formal calculation—though some is certainly required—and more on teaching students how to think critically with numerical or mathematical information. In the terminology adopted by MAA, AMATYC, and other mathematical organizations, students need to learn *quantitative reasoning* and to become *quantitatively literate*. There's been a recent rise in the popularity of quantitative reasoning courses for the non-STEM student. This book has been integral to the quantitative reasoning movement for years and continues to be at the forefront as an established entity designed to help you succeed in teaching quantitative reasoning to your students.

The Key to Success: A Context-Driven Approach

Broadly speaking, approaches to teaching mathematics can be divided into two categories:

- A *content-driven* approach is organized by mathematical ideas. After each mathematical topic is presented, examples of its applications are shown.

- A *context-driven* approach is organized by practical contexts. Applications drive the course, and mathematical ideas are presented as needed to support the applications.

The same content can be covered through either approach, but the context-driven approach has an enormous advantage: It motivates students by showing them directly how relevant mathematics is to their lives. In contrast, the content-driven approach tends to come across as "learn this content because it's good for you," causing many students to tune out before reaching the practical applications. For more details, see our article "General Education Mathematics: New Approaches for a New Millennium" (*AMATYC Review,* Fall 1999) or the discussion in the Epilogue of the book *Math for Life* by Jeffrey Bennett (Big Kid Science, 2014).

The Challenge: Winning Over Your Students

Perhaps the greatest challenge in teaching mathematics to students lies in winning them over—that is, convincing them that you have something useful to teach them. This challenge arises because by the time they reach college, many students dislike or fear mathematics. Indeed, the vast majority of students in general education mathematics courses are there not by choice, but because such courses are required for graduation. Reaching your students therefore requires that you teach with enthusiasm and convince them that mathematics is useful and enjoyable.

We've built this book around two important strategies that are designed to help you win students over:

- Confront negative attitudes about mathematics head on, showing students that their fear or loathing is ungrounded and that mathematics actually is relevant to their lives. This strategy is embodied in the Prologue of this book (pages P1–P13), which we urge you to emphasize in class. It continues implicitly throughout the rest of the text.
- Focus on goals that are meaningful to students—namely, on the goals of learning mathematics for *college, career*, and *life*. Your students will then learn mathematics because they will see how it affects their lives. This strategy forms the backbone of this book, as we have tried to build every unit around topics relevant to college, career, and life.

Modular Structure of the Book

Many of us would love to have a year or more to teach mathematics to general education students. Unfortunately, most schools have only a one-quarter or one-semester mathematics requirement, so we can cover only a fraction of the material we'd cover in an ideal course. This book is therefore organized with a modular structure that allows you to create a course to meet your (or your students') particular interests and constraints. The 12 chapters are organized broadly by contextual areas. Each chapter, in turn, is divided into a set of self-contained *units* that focus on particular concepts or applications. In most cases, you can cover chapters in any order; and while the units within each chapter build sequentially in terms of sophistication, in many cases you can skip certain units, particularly those toward the end of the chapter.

Prerequisite Mathematical Background

Because of its modular structure, this book can be used by students with a wide range of mathematical backgrounds. Many of the units require nothing more than arithmetic and a willingness to think about quantitative issues in new ways. Only a few units use techniques of algebra or geometry, and those skills are reviewed as they arise. This book should therefore be accessible to any student who has completed two or more years of high school mathematics. However, *this book is not remedial*: Although much of the book relies on mathematical techniques from secondary school, the techniques always arise in applications that students generally are not taught in high school and that require students to demonstrate their critical thinking skills.

For courses in which students do require more extensive prerequisite review, we have created a version of the *Using & Understanding Mathematics* MyMathLab course called *Using & Understanding Mathematics with Integrated Review* that includes just-in-time review of select topics where appropriate.

Changes in the Sixth Edition

We've been pleased by the positive responses of so many users to prior editions of this text. Nevertheless, a book that relies heavily on facts and data always requires a major updating effort to keep it current, and we are always looking for ways to improve clarity and pedagogy. As a result, users of prior editions will find many sections of this book to have been substantially revised or rewritten. Throughout the book we have added more examples and exercises pertaining to vocational careers, which should make the material more relevant to a wider variety of students. We have also made many other changes; while these are too many to list here, they include the following:

Chapter Openers Each chapter now opens with a multiple-choice question designed to illustrate an important way in which the chapter content connects with the book themes of *college*, *careers*, and *life*. These questions can spur lively in-class or online discussions.

Chapter 1 We significantly revised several units in Chapter 1. In particular, Unit 1A has been expanded to include a focus on evaluation of media information, and we rewrote portions of Units 1C and 1D to help students better understand and interpret Venn diagrams and tests of validity.

Chapter 2 We rewrote and reorganized Units 2A and 2B so basic ideas of units and systems of standardized units are now all covered in Unit 2A while Unit 2B focuses on more sophisticated problem solving with units.

Chapters 3 and 4 These two chapters contain several units that revolve around economic data—such as census data, the consumer price index, interest rates, taxes, and the federal budget—which obviously required major updates given the changes that have occurred in the U.S. economy in the four years since the previous edition was published.

Chapters 5 and 6 These chapters focus on statistical data, which means we updated or replaced large sections of the chapter content to reflect current data.

Chapter 7 We significantly revised the discussion of several key probability ideas to help students better understand them and overcome misconceptions.

Chapters 8 and 9 Units 8B, 8C, and 9C all rely heavily on population data, which means we revised significant portions of these units to reflect the 2010 U.S. Census and updated global demographic data.

Chapter 12 We significantly rewrote major portions of this chapter, particularly in Units 12A and 12C, both to update the political data and to clarify key concepts including those of preference schedules and redistricting.

Pedagogical Features

Besides the main narrative of the text, this book includes the following features, each designed with a specific pedagogical purpose in mind.

Chapter Overview Each chapter begins with a brief overview and a unit-by-unit listing of key content, designed both to show students how the chapter is organized and to help instructors decide which units to cover in class. It is then followed by a multiple-choice question designed to illustrate an important way in which the chapter content connects with the book themes of *college, careers,* and *life*.

Chapter Activity After the overview, each chapter offers an activity designed to spur student discussion of some interesting facet of the topics covered in the chapter. The activities may be done either individually or in small groups. An Activity Manual containing additional activities is available in the Tools for Success section of MyMathLab.

Time Out to Think Appearing throughout the book, the "Time Out to Think" features pose short conceptual questions designed to help students pause and reflect on important new ideas. They also serve as excellent starting points for class discussions and/or clicker questions.

Summary Boxes Flowing right along with the narrative are boxes that summarize key ideas, definitions, and formulas.

Examples and Case Studies Numbered examples are designed to build understanding and to offer practice with the types of questions that appear in the exercises. Each example is accompanied by a "Now Try" tag that relates the example to specific similar exercises. Occasional case studies go into more depth than the numbered examples.

In Your World These boxes focus on topics that students are likely to encounter in the world around them, whether in the news, in consumer decisions, or in political discussions. Examples include topics such as how to understand jewelry purchases, how to invest money in a sensible way, and how the chained consumer price index (CPI) differs from the standard CPI. This is further enhanced with a section of In Your World exercises in the exercise sets.

Brief Review This feature reviews key mathematical skills that students should have learned previously but in which many students still need review and practice. They appear in the book wherever a particular skill is first needed, and exercises based on the review boxes can be found at the end of the unit.

Using Technology These features give students clear instructions in the use of various technologies for computation, including scientific calculators, Microsoft Excel, and online technologies such as those built in to Google. Book-specific TI Tech Tips containing instructions for computations with a graphing calculator, such as the TI-83 and TI-84, are available in the Tools for Success section of MyMathLab.

Margin Features

- *By the Way* features contain interesting notes and asides relevant to the topic at hand.
- *Historical Note* remarks give historical context to the ideas presented in the chapter.
- *Technical Note* comments contain details that are important mathematically, but generally do not affect students' understanding of the material.

Mathematical Insight This feature builds upon mathematical ideas in the main narrative but goes somewhat beyond the level of other material in the book. Examples include boxes on the proof of the Pythagorean theorem, on Zeno's paradox, and on derivations of the financial formulas used for savings plans and mortgage loans.

Chapter Summary Appearing at the end of each chapter, the Chapter Summary offers a detailed outline of the chapter that students can use as a study guide.

Assessment Opportunities

Exercises are presented in various categories, making it easier for instructors to create assignments with a variety of problem types.

Quick Quiz This ten-question quiz appears at the end of each unit and allows students to check whether they understand key concepts before starting the exercise set. Note that students are asked not only to choose the correct multiple-choice answer but also to write a brief explanation of the reasoning behind their choice. Answers are included in the back of the text.

Review Questions Designed primarily for self-study, these questions ask students to summarize the important ideas covered in the unit and generally can be answered simply by reviewing the text.

Does It Make Sense? These qualitative questions test conceptual understanding by asking students to decide whether the given statements are sensible and to explain why or why not.

Basic Skills & Concepts These questions offer practice with the concepts covered in the unit. They can be used for homework assignments or for self-study (answers to most odd-numbered exercises appear in the back of the book). All of these questions are referenced by "Now Try" suggestions in the unit.

Further Applications Through additional applications, these exercises extend the ideas and techniques covered in the text.

In Your World These questions are designed to spur additional research or discussion that will help students relate the unit content to the book themes of college, careers, and life.

Using Technology These exercises, which support the Using Technology features, give students an opportunity to practice calculator or software skills introduced in the text.

Supplements

Student Supplements

Student's Study Guide and Solutions Manual

(ISBN 0-321-91532-1/978-0-321-91532-0)
James Lapp

- Includes detailed, worked-out solutions to the odd-numbered unit exercises.
- More than just a solutions manual, this supplement provides study tips and additional guidance.

Instructor Supplements

Instructor's Edition

(ISBN 0-321-91529-1/978-0-321-91529-0)

- Answers to all of the exercises and Quick Quizzes are included in the back of the book.

The following supplements are ONLINE ONLY and are available for download in the Pearson Higher Education catalog at www.pearsonhighered.com/irc or within your MyMathLab course.

Activity Manual

Shane Goodwin, *Brigham Young University–Idaho*, and Suzanne Topp, *Salt Lake Community College*

- More than 20 activities correlated to the textbook for those who wish to incorporate a more hands-on approach.
- Can be completed by students individually or in a group.
- Includes instructor notes with background information and discussion points.
- Available within MyMathLab.

Instructor's Solutions Manual

James Lapp

- Includes detailed, worked-out solutions to all of the exercises in the text.

Instructor's Testing Manual
Dawn Dabney

- Provides four alternative tests per chapter, including answer keys.

TestGen®

- Enables instructors to build, edit, print, and administer tests, using a computerized bank of questions developed to cover all the objectives of the text.
- Algorithmically based, allowing instructors to create multiple but equivalent versions of the same question or test with the click of a button.
- Tests can be printed or administered online.

PowerPoint® Lecture Presentation

- Classroom presentation slides.
- Includes lecture content and key graphics from the book.

MyMathLab & MathXL

MyMathLab® Online Course (access code required)
MyMathLab delivers **proven results** in helping individual students succeed.

- MyMathLab has a consistently positive impact on the quality of learning in higher education math instruction. MyMathLab can be successfully implemented in any environment—lab-based, hybrid, fully online, traditional—and demonstrates the quantifiable difference that integrated usage has on student retention, subsequent success, and overall achievement.
- MyMathLab's comprehensive online gradebook automatically tracks your students' results on tests, quizzes, homework, and in the study plan. You can use the gradebook to quickly intervene if your students have trouble, or to provide positive feedback on a job well done. The data within MyMathLab is easily exported to a variety of spreadsheet programs, such as Microsoft Excel. You can determine which points of data you want to export, and then analyze the results to determine success.

MyMathLab provides **engaging experiences** that personalize, stimulate, and measure learning for each student.

- **Personalized Learning:** MyMathLab offers several features that support adaptive learning: personalized homework and the adaptive study plan. These features allow your students to work on what they need to learn when it makes the most sense, maximizing their potential for understanding and success.
- **Exercises:** The homework and practice exercises in MyMathLab are correlated to the exercises in the textbook, and they regenerate algorithmically to give students unlimited opportunity for practice and mastery. The software offers immediate, helpful feedback when students enter incorrect answers.
- **Multimedia Learning Aids:** Exercises include guided solutions, sample problems, animations, videos, and eText access for extra help at point-of-use.
- **Expert Tutoring:** Although many students describe the whole of MyMathLab as "like having your own personal tutor," students using MyMathLab do have access to live tutoring from Pearson, from qualified math and statistics instructors.

And, MyMathLab comes from an **experienced partner** with educational expertise and an eye on the future.

- Knowing that you are using a Pearson product means knowing that you are using quality content. That means that our eTexts are accurate and our assessment tools work. It means we are committed to making MyMathLab as accessible as possible. MyMathLab exercises are compatible with the JAWS 12/13 screen reader, which enables multiple-choice and free-response problem-types to be read and interacted with via keyboard controls and math notation input. More information on this functionality is available at http://mymathlab.com/accessibility.
- Whether you are just getting started with MyMathLab, or have a question along the way, we're here to help you learn about our technologies and how to incorporate them into your course.

To learn more about how MyMathLab combines proven learning applications with powerful assessment, visit *www.mymathlab.com* or contact your Pearson representative.

Specific to the Using and Understanding Mathematics MyMathLab course:

- A new section of Getting Ready questions provides the ability to offer remediation for students who need it, similar to the Brief Reviews in the text.
- A question type applying the math concepts to a real-world situation using excerpts from current news articles.
- An Activity Manual correlated to the textbook contains additional activities that can be completed by students individually or in a group. Instructor notes with background information and discussion points are included.
- TI technology tips aligned with the location in the textbook in terms of the specific section and page number.
- Bonus unit on Mathematics and Business.

- Live RSS feeds from news sources such as ABC News are available so instructors and students have access to regular news updates. These articles can be used in class discussions and projects, as appropriate.

MyMathLab® Ready to Go Course (access code required) These new Ready to Go courses provide students with all the same great MyMathLab features, but make it easier for instructors to get started. Each course includes pre-assigned homework and quizzes to make creating a course even simpler. Ask your Pearson representative about the details for this particular course or to see a copy of this course.

Co-Requisite MyMathLab Course with Integrated Review (access code required) The co-requisite course integrates just-in-time review of developmental algebra throughout the college-level quantitative reasoning course.

MathXL® Online Course (access code required)
MathXL® is the homework and assessment engine that runs MyMathLab. (MyMathLab is MathXL plus a learning management system.)

With MathXL, instructors can:

- Create, edit, and assign online homework and tests using algorithmically generated exercises correlated at the objective level to the textbook.
- Create and assign their own online exercises and import TestGen tests for added flexibility.
- Maintain records of all student work tracked in MathXL's online gradebook.

With MathXL, students can:

- Take chapter tests in MathXL and receive personalized study plans and/or personalized homework assignments based on their test results.
- Use the study plan and/or the homework to link directly to tutorial exercises for the objectives they need to study.
- Access supplemental animations and video clips directly from selected exercises.

MathXL is available to qualified adopters. For more information, visit our website at www.mathxl.com, or contact your Pearson representative.

Acknowledgments

A textbook may carry author names, but it is the result of hard work by hundreds of committed individuals. This book has been under development for more than 25 years, and even its beginnings were a group effort, as one of the authors was a member of a committee at the University of Colorado that worked to establish one of the nation's first courses in quantitative reasoning. Since that beginning, the book has benefited from input and feedback from many faculty members and students.

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Prologue

LITERACY FOR THE MODERN WORLD

Equations are just the boring part of mathematics.

—Stephen Hawking, physicist

If you're like most students enrolled in a course using this text, you may think that your interests have relatively little to do with mathematics. But as the quote from Stephen Hawking indicates, mathematics is much more than equations, which is why this text will focus more on mathematical ideas and thinking. As you will see, this type of mathematical thinking is critical today for almost every career, as well as for the decisions and issues that we face daily as citizens in a modern technological society. In this Prologue, we'll discuss why mathematics is so important, why you may be better at it than you think, and how this course can provide you with the quantitative skills needed for your college courses, your career, and your life.

Q

Imagine that you're at a party and you've just struck up a conversation with a dynamic, successful lawyer. Which of the following are you most likely to hear her say during the course of your conversation?

- A "I really don't know how to read very well."
- B "I can't write a grammatically correct sentence."
- C "I'm awful at dealing with people."
- D "I've never been able to think logically."
- E "I'm bad at math."

A

We all know that the answer is E, because we've heard it so many times. Not just from lawyers, but from businessmen and businesswomen, actors and athletes, construction workers and sales clerks, and sometimes even teachers and CEOs. It would be difficult to imagine these same people admitting to any of choices A through D, but many people consider it socially acceptable to say that they are "bad at math." Unfortunately, this social acceptability comes with some very negative social consequences. You can probably think of a few already. For more, see the discussion under Misconception Seven on page P-7.

Job Satisfaction

Each chapter in this book will begin with an activity, which you may do individually or in groups. For this Prologue, we begin with an activity that will help you examine the role of mathematics in careers. Additional activities are available online in MyMathLab.

Top 20 Jobs for Job Satisfaction

1. Mathematician
2. Actuary (works with insurance statistics)
3. Statistician
4. Biologist
5. Software Engineer
6. Computer Systems Analyst
7. Historian
8. Sociologist
9. Industrial Designer
10. Accountant
11. Economist
12. Philosopher
13. Physicist
14. Parole Officer
15. Meteorologist
16. Medical Laboratory Technician
17. Paralegal Assistant
18. Computer Programmer
19. Motion Picture Editor
20. Astronomer

Source: JobsRated.com.

Everyone wants to find a career path that will bring lifelong job satisfaction, but what careers are most likely to do that? A recent survey evaluated 200 different jobs according to five criteria: salary, long-term employment outlook, work environment, physical demands, and stress. The table to the left shows the top 20 jobs according to this survey. Notice that most of the top 20 jobs require mathematical skills, and all of them require an ability to reason with quantitative information.

You and your classmates can conduct your own smaller study of job satisfaction. There are many ways to do this, but here is one procedure you might try:

1. Each of you should identify at least three people with full-time jobs to interview briefly. You may choose parents, friends, acquaintances, or just someone whose job interests you.
2. Identify an appropriate job category for each interviewee (similar to the categories in the table to the left). Ask each interviewee to rate his or her job on a scale of 1 (worst) to 5 (best) on each of the five criteria: salary, long-term employment outlook, work environment, physical demands, and stress. You can then add the ratings for the five criteria to come up with a total "job satisfaction" rating for each job.
3. Working together as a class, compile the data to rank all the jobs. Show the final results in a table that ranks the jobs in order of job satisfaction.
4. Discuss the results. Are they consistent with the survey results shown in the table? Do they surprise you in any way? Will they have any effect on your own career plans?

What Is Quantitative Reasoning?

Literacy is the ability to read and write, and it comes in varying degrees. Some people can recognize only a few words and write only their names; others read and write in many languages. A primary goal of our educational system is to provide citizens with a level of literacy sufficient to read, write, and reason about the important issues of our time.

Today, the abilities to interpret and reason with **quantitative information**—information that involves mathematical ideas or numbers—are crucial aspects of literacy. These abilities, often called **quantitative reasoning** or **quantitative literacy**, are essential to understanding issues that appear in the news every day. The purpose of this book is to help you gain skills in quantitative reasoning as it applies to issues you will encounter in

- your subsequent coursework,
- your career, and
- your daily life.

Quantitative Reasoning and Culture

Quantitative reasoning enriches the appreciation of both ancient and modern culture. The historical record shows that nearly all cultures devoted substantial energy to mathematics and to science (or to observational studies that predated modern science). Without a sense of how quantitative concepts are used in art, architecture, and science, you cannot fully appreciate the incredible achievements of the Mayans in Central America, the builders of the great city of Zimbabwe in Africa, the ancient Egyptians and Greeks, the early Polynesian sailors, and countless others.

Similarly, quantitative concepts can help you understand and appreciate the works of the great artists. Mathematical concepts play a major role in everything from the work of Renaissance artists like Leonardo da Vinci and Michelangelo to the pop culture of television shows like *The Big Bang Theory*. Other ties between mathematics and the arts can be found in both modern and classical music, as well as in the digital production of music. Indeed, it is hard to find popular works of art, film, or literature that do not rely on mathematics in some way.

Mathematics knows no races or geographic boundaries; for mathematics, the cultural world is one country.

—David Hilbert (1862–1943), mathematician

Quantitative Reasoning in the Work Force

Quantitative reasoning is important in the work force. A lack of quantitative skills puts many of the most challenging and highest-paying jobs out of reach. Table P.1 defines skill levels in language and mathematics on a scale of 1 to 6, and Table P.2 shows the typical levels needed in many jobs.

Note that the occupations requiring high skill levels are generally the most prestigious and highest paying. Note also that most of these occupations call for high skill levels in *both* language and math, refuting the myth that if you're good at language you don't have to be good at mathematics, and vice versa.

TABLE P.1 Skill Levels

Level	Language Skills	Math Skills
1	Recognizes 2500 two- or three-syllable words. Reads at a rate of 95–120 words per minute. Writes and speaks simple sentences.	Adds and subtracts two-digit numbers. Does simple calculations with money, volume, length, and weight.
2	Recognizes 5000–6000 words. Reads 190–215 words per minute. Reads adventure stories and comic books, as well as instructions for assembling model cars. Writes compound and complex sentences with proper grammar and punctuation.	Adds, subtracts, multiplies, and divides all units of measure. Computes ratio, rate, and percentage. Draws and interprets bar graphs.
3	Reads novels and magazines, as well as safety rules and equipment instructions. Writes reports with proper format and punctuation. Speaks well before an audience.	Understands basic geometry and algebra. Calculates discount, interest, profit and loss, markup, and commissions.
4	Reads novels, poems, newspapers, and manuals. Prepares business letters, summaries, and reports. Participates in panel discussions and debates. Speaks extemporaneously on a variety of subjects.	Has true quantitative reasoning abilities. Understands logic, problem solving, ideas of statistics and probability, and modeling.
5	Reads literature, book and play reviews, scientific and technical journals, financial reports, and legal documents. Can write editorials, speeches, and critiques.	Knows calculus and statistics. Is able to deal with econometrics.
6	Same types of skills as level 5, but more advanced.	Works with advanced calculus, modern algebra, and statistics.

Source: Data from the *Wall Street Journal.*

TABLE P. 2 Skill-Level Requirements

Occupation	Language Level	Math Level	Occupation	Language Level	Math Level
Biochemist	6	6	Web page designer	5	4
Computer engineer	6	6	Corporate executive	5	5
Mathematician	6	6	Computer sales agent	4	4
Cardiologist	6	5	Athlete's agent	4	4
Social psychologist	6	5	Management trainee	4	4
Lawyer	6	4	Insurance sales agent	4	4
Tax attorney	6	4	Retail store manager	4	4
Newspaper editor	6	4	Cement mason	3	3
Accountant	5	5	Poultry farmer	3	3
Personnel manager	5	4	Tile setter	3	3
Corporate president	5	5	Travel agent	3	3
Weather forecaster	5	5	Janitor	3	2
Secondary teacher	5	5	Short-order cook	3	2
Elementary teacher	5	4	Assembly-line worker	2	2
Financial analyst	5	5	Toll collector	2	2
Journalist	5	4	Laundry worker	1	1

Source: Data from the *Wall Street Journal*.

Misconceptions about Mathematics

Do you consider yourself to have "math phobia" (fear of mathematics) or "math loathing" (dislike of mathematics)? We hope not—but if you do, you aren't alone. Many adults harbor fear or loathing of mathematics, and unfortunately, these attitudes are often reinforced by classes that present mathematics as an obscure and sterile subject.

In reality, mathematics is not nearly so dry as it sometimes seems in school. Indeed, attitudes toward mathematics often are directed not at what mathematics really is but at some common misconceptions about mathematics. Let's investigate a few of these misconceptions and the reality behind them.

Misconception One: Math Requires a Special Brain

One of the most pervasive misconceptions is that some people just aren't good at mathematics because learning mathematics requires special or rare abilities. The reality is that nearly everyone can do mathematics. All it takes is self-confidence and hard work—the same qualities needed to learn to read, to master a musical instrument, or to become skilled at a sport. Indeed, the belief that mathematics requires special talent found in a few elite people is peculiar to the United States. In other countries, particularly in Europe and Asia, *all* students are expected to become proficient in mathematics.

Of course, different people learn mathematics at different rates and in different ways. For example, some people learn by concentrating on concrete problems, others by thinking visually, and still others by thinking abstractly. No matter what type of thinking style you prefer, you can succeed in mathematics.

We are all mathematicians… [your] forte lies in navigating the complexities of social networks, weighing passions against histories, calculating reactions, and generally managing a system of information that, when all laid out, would boggle a computer.

—A. K. Dewdney, 200% of Nothing

Misconception Two: The Math in Modern Issues Is Too Complex

Some people claim that the advanced mathematical concepts underlying many modern issues are too complex for the average person to understand. It is true that only a few people receive the training needed to work with or discover advanced mathematical concepts. However, most people are capable of understanding enough about the mathematical basis of important issues to develop informed and reasoned opinions.

The situation is similar in other fields. For example, years of study and practice are required to become a proficient professional writer, but most people can read a book. It takes hard work and a law degree to become a lawyer, but most people can understand how the law affects them. And though few have the musical talent of Mozart, anyone can learn to appreciate his music. Mathematics is no different. If you've made it this far in school, you can understand enough mathematics to succeed as an individual and a concerned citizen.

Skills are to mathematics what scales are to music or spelling is to writing. The objective of learning is to write, to play music, or to solve problems—not just to master skills.

—from Everybody Counts, a report of the National Research Council

Misconception Three: Math Makes You Less Sensitive

Some people believe that learning mathematics will somehow make them less sensitive to the romantic and aesthetic aspects of life. In fact, understanding the mathematics that explains the colors of a sunset or the geometric beauty in a work of art can only enhance aesthetic appreciation. Furthermore, many people find beauty and elegance in mathematics itself. It's no accident that people trained in mathematics have made important contributions to art, music, and many other fields.

It is impossible to be a mathematician without being a poet in the soul.

—Sophia Kovalevskaya (1850–1891), Russian mathematician

Misconception Four: Math Makes No Allowance for Creativity

The "turn the crank" nature of the problems in many textbooks may give the impression that mathematics stifles creativity. Some of the facts, formalisms, and skills required for mathematical proficiency are fairly cut and dried, but *using* these mathematical tools takes creativity. Consider designing and building a home. The task demands specific skills to lay the foundation, frame in the structure, install plumbing and wiring, and paint walls. But building the home involves much more: Creativity is needed to develop the architectural design, respond to on-the-spot problems during construction, and factor in constraints based on budgets and building codes. The mathematical skills you've learned in school are like the skills of carpentry or plumbing. Applying mathematics is like the creative process of building a home.

Tell me, and I will forget. Show me, and I may remember. Involve me, and I will understand.

—Confucius (c. 551–479 B.C.)

IN YOUR WORLD

People Who Studied Mathematics

The critical thinking skills developed through the study of mathematics are valuable in many careers. The following is only a small sample of people who studied mathematics but became famous for work in other fields. Many of the names come from "Famous Nonmathematicians," a list compiled by Steven G. Buyske, Rutgers University.

Ralph Abernathy, civil rights leader, BS in mathematics, Alabama State University

Corazon Aquino, former president of the Philippines, a mathematics minor

Mayim Bialik, actress on *The Big Bang Theory*, studied mathematics as part of her Ph.D. in neuroscience

Harry Blackmun, former Supreme Court justice, summa cum laude in mathematics, Harvard University

James Cameron, film director, studied physics before leaving college, works in oceanic and space research

Lewis Carroll (Charles Dodgson), author of *Alice in Wonderland,* a mathematician

David Dinkins, former mayor of New York City, BA in mathematics, Howard University

Alberto Fujimori, former president of Peru, MS in mathematics, University of Wisconsin

Art Garfunkel, musician, MA in mathematics, Columbia University

Grace Hopper, computer pioneer and first woman Rear Admiral in the U.S. Navy, Ph.D. in mathematics, Yale University

Mae Jemison, first African-American woman in space, studied mathematics as part of her degree in chemical engineering from Stanford University

John Maynard Keynes, economist, MA in mathematics, Cambridge University

Hedy Lamarr, actress called "the most beautiful woman in Hollywood," invented and patented the mathematical technique of "frequency hopping"

Lee Hsien Loong, politician in Singapore, BA in mathematics, Cambridge University

Brian May, lead guitarist for the band Queen, completed his Ph.D. in astrophysics in 2007, Imperial College

Danica McKellar, actress, BA with highest honors in mathematics, UCLA, and co-discoverer of the Chayes-McKellar-Winn theorem

Edwin Moses, three-time Olympic champion in the 400-meter hurdles, studied mathematics as part of his degree in physics from Morehouse College

Florence Nightingale, pioneer in nursing, studied mathematics and applied it to her work

Natalie Portman, Oscar-winning actress, semifinalist in Intel Science Talent Search and co-author of two published scientific papers

Sally Ride, first American woman in space, studied mathematics as part of her Ph.D. in physics from Stanford University

David Robinson, basketball star, bachelor's degree in mathematics, U.S. Naval Academy

Alexander Solzhenitsyn, Nobel prize–winning Russian author, degrees in mathematics and physics from the University of Rostov

Bram Stoker, author of *Dracula,* studied mathematics at Trinity University, Dublin

Laurence Tribe, Harvard law professor, summa cum laude in mathematics, Harvard University

Virginia Wade, Wimbledon champion, bachelor's degree in mathematics, Sussex University

Misconception Five: Math Provides Exact Answers

A mathematical formula will yield a specific result, and in school that result may be marked right or wrong. But when you use mathematics in real-life situations, answers are never so clear cut. For example:

> *A bank offers simple interest of 3%, paid at the end of one year (that is, after one year the bank pays you 3% of your account balance). If you deposit $1000 today and make no further deposits or withdrawals, how much will you have in your account after one year?*

A straight mathematical calculation seems simple enough: 3% of $1000 is $30; so you should have $1030 at the end of a year. But will you? How will your balance be affected by service charges or taxes on interest earned? What if the bank fails? What if the bank is located in a country in which the currency collapses during the year? Choosing a bank in which to invest your money is a *real* mathematics problem that doesn't necessarily have a simple or definitive solution.

Probably the most harmful misconception is that mathematics is essentially a matter of computation. Believing this is roughly equivalent to believing that writing essays is the same as typing them.

—John Allen Paulos, mathematician

Misconception Six: Math Is Irrelevant to My Life

No matter what your path in college, career, and life, you will find mathematics involved in many ways. A major goal of this text is to show you hundreds of examples in which mathematics applies to everyone's life. We hope you will find that mathematics is not only relevant but also interesting and enjoyable.

Neglect of mathematics works injury to all knowledge…

—Roger Bacon (1214–1294), English philosopher

Misconception Seven: It's OK to Be "Bad at Math"

For our final misconception, let's return to the multiple-choice question in the opening of this Prologue. You'll not only hear many otherwise intelligent people say "I'm bad at math," but it's sometimes said almost as a point of pride, with no hint of embarrassment. Yet the statement often isn't even true. Our successful lawyer, for example, almost certainly did well in all subjects in school, including math, so she is more likely expressing an attitude than a reality.

Unfortunately, this type of attitude can cause a lot of damage. Mathematics underlies nearly everything in modern society, from the daily financial decisions that all of us must make to the way in which we understand and approach global issues of the economy, politics, and science. We cannot possibly hope to act wisely if we approach mathematical ideas with a poor attitude. Moreover, it's an attitude that can easily spread to others. After all, if a child hears a respected adult saying that he or she is "bad at math," the child may be less inspired to do well.

So before you begin your coursework, think about your own attitudes toward mathematics. There's no reason why anyone should be "bad at math" and every reason to develop skills of mathematical thinking. With a good attitude and some hard work, by the end of your course you'll not only be better at math, but you'll be helping future generations by making it socially unacceptable for anyone to be "bad at math."

You must be the change you wish to see in the world.

—Mahatma Ghandi (1869–1948)

What *Is* Mathematics?

In discussing misconceptions, we identified what mathematics is *not*. Now let's look at what mathematics *is*. The word *mathematics* is derived from the Greek word *mathematikos,* which means "inclined to learn." Literally speaking, to be mathematical is to be curious, open-minded, and interested in always learning more! Today, we tend to look at mathematics in three different ways: as the sum of its branches, as a way to model the world, and as a language.

Mathematics as the Sum of Its Branches

As you progressed through school, you probably learned to associate mathematics with some of its branches. Among the better known branches of mathematics are these:

- **logic**—the study of principles of reasoning;
- **arithmetic**—methods for operating on numbers;
- **algebra**—methods for working with unknown quantities;
- **geometry**—the study of size and shape;
- **trigonometry**—the study of triangles and their uses;
- **probability**—the study of chance;
- **statistics**—methods for analyzing data; and
- **calculus**—the study of quantities that change.

One can view mathematics as the sum of its branches, but in this book we'll focus on how different branches of mathematics support the more general goals of quantitative thinking and critical reasoning.

Mathematics as a Way to Model the World

Mathematics also may be viewed as a tool for creating models, or representations of real phenomena. Modeling is not unique to mathematics. For example, a road map is a model that represents the roads in some region.

Mathematical models can be as simple as a single equation that predicts how the money in your bank account will grow or as complex as a set of thousands of interrelated equations and parameters used to represent the global climate. By studying models, we gain insight into otherwise unmanageable problems. A global climate model, for example, can help us understand weather systems and ask "what if" questions about how human activity may affect the climate. When a model is used to make a prediction that does *not* come true, it points out areas where further research is needed. Today, mathematical modeling is used in nearly every field of study. Figure P.1 indicates some of the many disciplines that use mathematical modeling to solve problems.

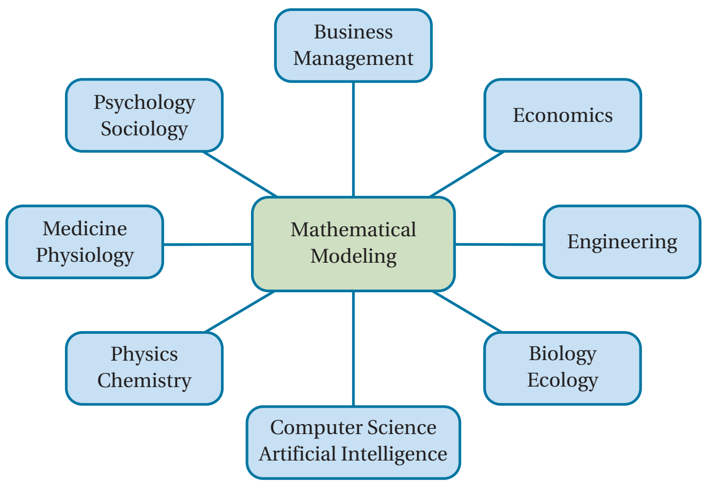


FIGURE P.1

Mathematics as a Language

A third way to look at mathematics is as a language with its own vocabulary and grammar. Indeed, mathematics often is called "the language of nature" because it is so useful for modeling the natural world. As in any language, different degrees of fluency are possible. From this point of view, quantitative literacy is the level of fluency required for success in today's world.

The idea of mathematics as a language also is useful in thinking about how to *learn* mathematics. Table P.3 compares learning mathematics to learning a language and learning art.

The Book of Nature is written in the language of mathematics.

—Galileo

TABLE P.3 Learning Mathematics: An Analogy to Language and Art

Learning a Language	Learning the Language of Art	Learning the Language of Mathematics
Learn many styles of speaking and writing, such as essays, poetry, and drama.	Learn many styles of art, such as classical, renaissance, impressionist, and modern.	Learn techniques from many branches of mathematics, such as arithmetic, algebra, and geometry.
Place literature in context through the history and social conditions under which it was created.	Place art in context through the history and social conditions under which it was created.	Place mathematics in context through its history, purposes, and applications.
Learn the elements of language—such as words, parts of speech (nouns, verbs, etc.), and rules of grammar—and practice their proper use.	Learn the elements of visual form— such as lines, shapes, colors, and textures—and practice using them in your own art work.	Learn the elements of mathematics—such as numbers, variables, and operations—and practice using them to solve simple problems.
Critically analyze language in forms such as novels, short stories, essays, poems, speeches, and debates.	Critically analyze works of art including painting, sculpture, architecture, and photography.	Critically analyze quantitative information in mathematical models, statistical studies, economic forecasts, investment strategies, and more.
Use language creatively for your own purposes, such as writing a term paper or story or engaging in debate.	Use your sense of art creatively, such as in designing your house, taking a photograph, or making a sculpture.	Use mathematics creatively to solve problems you encounter and to help you understand issues in the modern world.

How to Succeed in Mathematics

If you are reading this book, you probably are enrolled in a mathematics course. The keys to success in your course include approaching the material with an open and optimistic frame of mind, paying close attention to how useful and enjoyable mathematics can be in *your* life, and studying effectively and efficiently. The following sections offer a few specific hints that may be of use as you study.

Using This Book

Before we get into more general strategies for studying, here are a few guidelines that will help you use *this* book most effectively.

- Before doing any assigned exercises, read assigned material *twice*:
  - On the first pass, read quickly to gain a "feel" for the material and concepts presented.
  - On the second pass, read the material in more depth and work through the examples carefully.

- During the second reading, take notes that will help you when you go back to study later. In particular:
  - Use the margins! The wide margins in this textbook are designed to give you plenty of room to make notes as you study.
  - Don't highlight—underline! Using a pen or pencil to underline material requires greater care than highlighting and therefore helps to keep you alert as you study.
- After you complete the reading, and again when studying for exams, make sure you can answer the Quick Quiz and Review Questions at the end of each unit.
- You'll learn best by *doing,* so do plenty of the end-of-unit exercises. In particular, try some of the exercises that have answers in the back of the book, in addition to those assigned by your instructor.
- If you have access to MyMathLab with this book, be sure to take advantage of the many additional study resources available on this website.

Budgeting Your Time

The single most important key to success in any college course is to spend enough time studying. A general rule of thumb for college classes is that you should expect to study about 2 to 3 hours per week *outside* class for each unit of credit. For example, a student taking 15 credit hours should spend 30 to 45 hours each week studying outside of class. Combined with time in class, this works out to a total of 45 to 60 hours per week—not much more than the time required of a typical job, and you get to choose your own hours. Of course, if you are working or taking care of a family while you attend school, you will need to budget your time carefully.

The following table gives some rough guidelines for how you might divide your studying time in your mathematics course. If you are spending fewer hours than these guidelines suggest, you could probably improve your grade by studying more. If you are spending more hours than these guidelines suggest, you may be studying inefficiently; in that case, you should talk to your instructor about how to study more effectively.

If Your Course Is	Time for Reading the Assigned Text (per Week)	Time for Homework Assignments (per Week)	Time for Review and Test Preparation (Average per Week)	Total Study Time (per Week)
3 credits	1 to 2 hours	3 to 5 hours	2 hours	6 to 9 hours
4 credits	2 to 3 hours	3 to 6 hours	3 hours	8 to 12 hours
5 credits	2 to 4 hours	4 to 7 hours	4 hours	10 to 15 hours

General Strategies for Studying

- Budget your time effectively. One or two hours each day is more effective, and far less painful, than studying all night before homework is due or before exams.
- Engage your brain. Learning is an active process, not a passive experience. Whether you are reading, listening to a lecture, or working on assignments, always make sure that your mind is actively engaged. If you find your mind drifting or falling asleep, make a conscious effort to revive yourself or take a break if necessary.

- Don't miss class. Listening to lectures and participating in class activities and discussions are much more effective than reading someone else's notes. Active participation will help you retain what you are learning.
- Be sure to complete any assigned reading *before* the class in which it will be discussed. This is crucial because class lectures and discussions are designed to help reinforce key ideas from the reading.
- Start your homework early. The more time you allow yourself, the easier it is to get help if you need it. If a concept gives you trouble, first try additional reading or studying beyond what has been assigned. If you still have trouble, ask for help: You surely can find friends, peers, or teachers who will help you learn.
- Working together with friends can be valuable in helping you understand difficult concepts. However, be sure that you learn *with* your friends and do not become dependent on them.
- Don't try to multitask. A large body of research shows that human beings simply are not good at multitasking: When we attempt it, we do more poorly at all of the individual tasks. And in case you think you are an exception, the same research found that those people who believed they were best at multitasking were actually the worst! When it is time to study, turn off your electronic devices, find a quiet spot, and give your work a focused effort of concentration.

Preparing for Exams

- Rework exercises and other assignments. Try additional exercises to be sure you understand the concepts. Study your assignments, quizzes, and exams from earlier in the semester.
- Study your notes from lectures and discussions, and reread relevant sections in your textbook. Pay attention to what your instructor expects you to know for an exam.
- Study individually *before* joining a study group with friends. Study groups are effective only if *every* individual comes prepared to contribute.
- Don't stay up too late before an exam. Don't eat a big meal within an hour of the exam (thinking is more difficult when blood is going to the digestive system).
- Try to relax before and during the exam. If you have studied effectively, you are capable of doing well. Staying relaxed will help you think clearly.

Presenting Homework and Writing Assignments

All work that you turn in should be of *collegiate* quality: neat and easy to read, well organized, and demonstrating mastery of the subject matter. Future employers and teachers will expect this quality of work. Moreover, although submitting homework of collegiate quality requires "extra" effort, it serves two important purposes directly related to learning:

1. The effort you expend in clearly explaining your work solidifies your understanding. In particular, research has shown that writing and speaking trigger different areas of your brain. By writing something down—even when you think you already understand it—your learning is reinforced by involving other areas of your brain.
2. By making your work clear and self-contained (that is, making it a document that you can read without referring to the questions in the text), you will have a much more useful study guide when you review for a quiz or exam.

The following guidelines will help ensure that your assignments meet the standards of collegiate quality:

- Always use proper grammar, proper sentence and paragraph structure, and proper spelling. Do not use texting shorthand.
- All answers and other writing should be fully self-contained. A good test is to imagine that a friend is reading your work and to ask yourself whether the friend would understand exactly what you are trying to say. It is also helpful to read your work out loud to yourself, making sure that it sounds clear and coherent.
- In problems that require calculation:
  - Be sure to *show your work* clearly. By doing so, both you and your instructor can follow the process you used to obtain an answer. Also, please use standard mathematical symbols, rather than "calculator-ese." For example, show multiplication with the × symbol (not with an asterisk), and write 10^5, not 10^5 or 10E5.
  - *Word problems should have word answers*. That is, after you have completed any necessary calculations, any problem stated in words should be answered with one or more *complete sentences* that describe the point of the problem and the meaning of your solution.
  - Express your word answers in a way that would be *meaningful* to most people. For example, most people would find it more meaningful if you express a result of 720 hours as 1 month. Similarly, if a precise calculation yields an answer of 9,745,600 years, it may be more meaningful in words as "nearly 10 million years."
- Include illustrations whenever they help explain your answer, and make sure your illustrations are neat and clear. For example, if you graph by hand, use a ruler to make straight lines. If you use software to make illustrations, be careful not to make them overly cluttered with unnecessary features.
- If you study with friends, be sure that you turn in your own work stated in your own words—you should avoid anything that might even give the *appearance* of possible academic dishonesty.

Prologue

DISCUSSION QUESTIONS

1. **Mathematics in Modern Issues.** Describe at least one way that mathematics is involved in each issue below.

   Example: The spread of AIDS: Mathematics is used to study the probability of contracting AIDS.

   a. The long-term viability of the Social Security system
   b. The appropriate level for the federal gasoline tax
   c. National health care policy
   d. Job discrimination against women or ethnic minorities
   e. Effects of population growth (or decline) on your community
   f. Possible bias in standardized tests (e.g., the SAT)
   g. The degree of risk posed by carbon dioxide emissions
   h. Immigration policy of the United States
   i. Violence in public schools
   j. Whether certain types of guns or ammunition should be banned
   k. An issue of your choice from today's news

2. **Quantitative Concepts in the News.** Identify the major unresolved issue discussed in today's news. List at least three areas in which quantitative concepts play a role in the policy considerations of this issue.

3. **Mathematics and the Arts.** Choose a well-known historical figure in a field of art in which you have a personal interest (e.g., a painter, sculptor, musician, or architect). Briefly describe how mathematics played a role in or influenced that person's work.

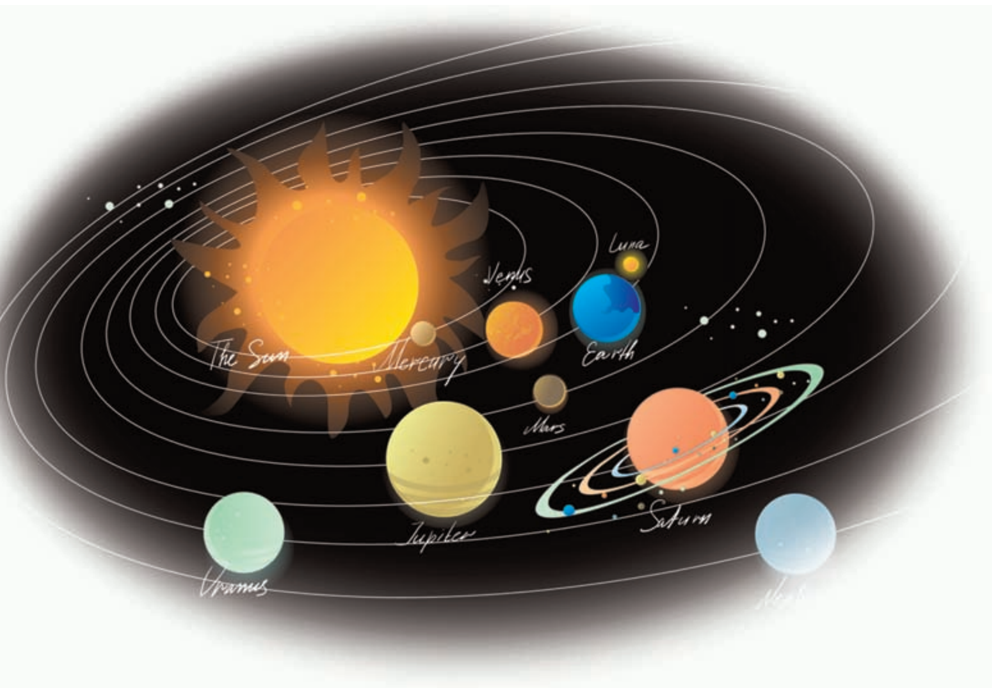
4. **Quantitative Literature.** Choose a favorite work of literature (poem, play, short story, or novel). Describe one or more instances in which quantitative reasoning is helpful in understanding the subtleties intended by the author.

5. **Your Quantitative Major.** Identify ways in which quantitative reasoning is important within your major field of study. (If you haven't yet chosen a major, pick a field that you are considering for your major.)
6. **Career Preparation.** Realizing that most Americans change careers several times during their lives, identify at least three occupations in Table 2 that interest you. Do you have the necessary skills for them at this time? If not, how can you acquire these skills?
7. **Attitudes Toward Mathematics.** What is your attitude toward mathematics? If you have a negative attitude, can you identify when that attitude developed? If you have a positive attitude, can you explain why? How might you encourage someone with a negative attitude to become more positive?
8. **"Bad at Math" as a Social Disease.** Discuss reasons why many people think being "bad at math" is socially acceptable and how we as a society can change those attitudes. If you were a teacher, what would you do to ensure that your students develop positive attitudes toward mathematics?

1 THINKING CRITICALLY

The primary goal of this text is to help you develop the quantitative reasoning skills you will need to succeed in other college courses, in your career, and in your life as a citizen in an increasingly complex world. Quantitative reasoning combines basic mathematical skills—most of which you already have—with the ability to approach problems in a critical and analytical way. For this reason, we devote this first chapter to studying ideas of logic that will develop your ability to think critically.

Q **Perhaps you, like millions of others, have received this message: "On August 27, Mars will look as large and bright as the full Moon. Don't miss it, because no one alive today will ever see this again." This claim:**



- **A** is true, because on this date Mars will be closer to Earth than any time in thousands of years.
- **B** is true, because on this date Mars will be closer to Earth than the Moon.
- **C** was true for the year 2012, but not for other years.
- **D** is false.
- **E** is partially true: Mars really will be this bright, but it happens every year on August 27, so you'll see it again.

Mathematics is just logic with numbers attached.

—Marilyn vos Savant, American author

A If you're like most students, you may be wondering what this question has to do with math. The answer is "a lot." To begin with, logic is actually a branch of mathematics, and you can use logic to analyze the claim about Mars. Beyond that, the question also involves mathematics on several deeper levels. For example, the statement "Mars will look as large … as the full Moon" is a statement about *angular size*, which is a mathematical way of expressing how large an object appears to your eye. In addition, a full understanding of the claim requires understanding how the Moon orbits Earth and planets orbit the Sun, which means understanding that orbits have the mathematical shape called an *ellipse* and obey precise mathematical laws.

So what's the answer? Here's a key hint: Think about the fact that Mars is a planet orbiting the Sun while the Moon orbits Earth. Given that fact, ask yourself when, if ever, Mars could appear as large and bright as the full Moon. To see the answer and discussion, go to Example 11 on Page 10.

UNIT 1A

Living in the Media Age: Explore common fallacies, or deceptive arguments, and learn how to avoid them.

UNIT 1B

Propositions and Truth Values: Study basic components of logic, including propositions, truth values, truth tables, and the logical connectors *and, or,* and *if … then.*

UNIT 1C

Sets and Venn Diagrams: Understand sets, and use Venn diagrams to visualize relationships among sets.

UNIT 1D

Analyzing Arguments: Learn to distinguish and evaluate basic inductive and deductive arguments.

UNIT 1E

Critical Thinking in Everyday Life: Apply logic to common situations in everyday life.

Bursting Bubble

Use this activity to gain a sense of the kinds of problems this chapter will enable you to study. Additional activities are available online in MyMathLab.

The global economy is still recovering from the deep recession and financial crisis that began in 2007 and led to massive bank bailouts, huge increases in unemployment, and many other severe economic consequences. While the recession had many causes, the clear trigger that set it off was a fairly sudden collapse in housing prices. This collapse led many homeowners to default on their home mortgages, which in turn created a crisis for banks and other institutions that bought, sold, or insured home mortgages. If we hope to avoid similar crises in the future, a key question is whether there were early warning signs that might have allowed both individuals and policy makers to make decisions that could have prevented the problems before they occurred.

Figure 1.A shows how average (median) home prices have compared to average income over the past several decades. A ratio of 3.0, for example, means that the average home price is three times the average annual household income of Americans; that is, if you had a household income of $50,000 per year and bought an average house, the price of your house would be $150,000. Notice that the ratio remained below about 3.5 until 2001, when it suddenly started shooting up, which is why the period from 2001 to about 2006 is said to have been marked by a *housing bubble*.

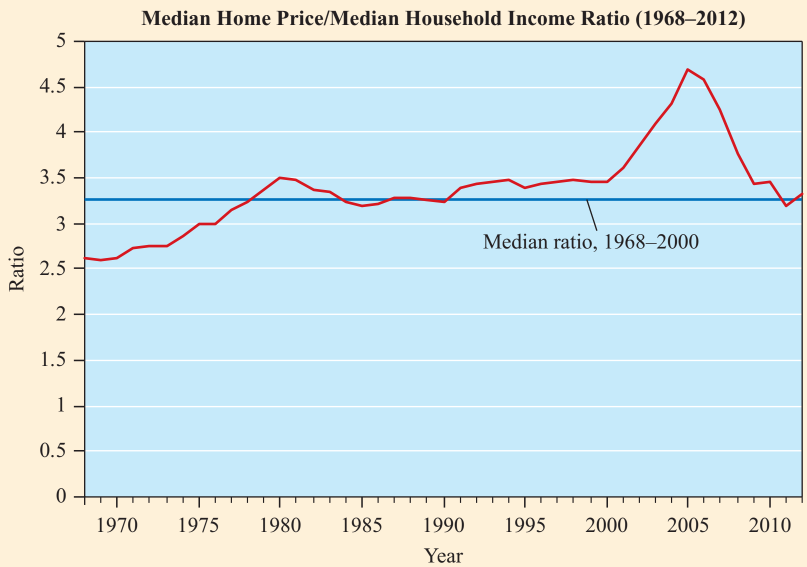


FIGURE 1.A *Source:* Data from *The State of the Nation's Housing 2013*, used with permission from the Joint Center for Housing Studies of Harvard University. All rights reserved.

Was the change in the home price to income ratio a warning sign that should have been heeded? Use your powers of logic—the topic of this chapter—to discuss the following questions.

1. Consider a family with an annual income of $50,000. If they bought an average home, how much would they have spent in 2000, when the home price to income ratio was about 3.5? How much would they have spent in 2005, when the ratio was about 4.7?
2. In percentage terms, a rise in the ratio from 3.5 to 4.7 is an increase of nearly 35%. Because the ratio was *below* 3.5 for decades before 2001, we can conclude that the average home

was at least 35% more expensive relative to income in 2005 than it had been historically. What can you infer about how the percentage of income that a family spent on housing changed during the housing bubble?

3. In general, a family can increase the percentage of its income that it spends on housing only if some combination of the following three things happens: (1) its income increases, so it can afford to spend more of it on housing; (2) it cuts expenses in other areas; or (3) it borrows more money. Based on your understanding of the housing crisis, what happened in most cases during the housing bubble?
4. Overall, do you think it was inevitable that the bubble would burst? Why or why not?
5. How could you use the data on the home price to income ratio to help *you* make a decision about how much to spend when you are looking to buy a home?
6. Bonus: As home prices rose during the bubble, the optimists claimed that the higher prices could be sustained. Do a bit of Web research to learn how they justified this belief. Do you think their arguments sounded reasonable at the time? Do they still sound reasonable with hindsight?
7. Additional Research: The data shown here reflect a nationwide average, but the home price to income ratio varies considerably in different cities and regions. Find data for a few different cities or regions, and discuss the differences.

UNIT 1A Living in the Media Age

We are living in what is sometimes called the "media age," because we are in almost constant contact with media of some sort. Some of the media content is printed in books, newspapers, magazines, and billboards. Much more is delivered electronically through the Internet, tablets and smart phones, television, movies, and more. Most people rely on these media sources for information, which means they form opinions and beliefs based on these same sources.

Unfortunately, much of the information in the media is either inaccurate or biased, designed less to inform us than to convince us of something that may or may not be true. As a result, the only way to make sense of the media information bombardment is to equip yourself with an understanding of the ways in which people try to manipulate your views. In this first unit, we'll explore a few of the tools that can help you navigate the media intelligently. These tools will also provide a foundation for the critical thinking and quantitative reasoning that we'll focus on in the rest of this book.

The Concept of Logical Argument

If you read the comments that follow many news articles on the Web, you'll often see heated discussions that might look much like this "argument" between two classmates.

Ethan: *The death penalty is immoral.*
Jessica: *No it isn't.*
Ethan: *Yes it is! Judges who give the death penalty should be impeached.*
Jessica: *You don't even know how the death penalty is decided.*

People generally quarrel because they cannot argue.

—*G. K. Chesterton (1874–1936), English author*

Ethan: *I know a lot more than you know!*

Jessica: *I can't talk to you; you're an idiot!*

This type of argument may be common, but it accomplishes little. It doesn't give either person insight into the other's thinking, and it is unlikely to change either person's opinion. Fortunately, there is a better way to argue. We can use skills of **logic**—the study of the methods and principles of reasoning. Arguing logically may still not change either person's position, but it can help them understand each other.

In logic, the term **argument** refers to a reasoned or thoughtful process. Specifically, an argument uses a set of facts or assumptions, called **premises**, to support a **conclusion.** Some arguments provide strong support for their conclusions, but others do not. An argument that fails to make a compelling case for its conclusion may contain some error in reasoning, or **fallacy** (from the Latin for "deceit" or "trick"). In other words, a fallacious argument tries to persuade in a way that doesn't really make sense when analyzed carefully.

Definitions

Logic is the study of the methods and principles of reasoning.

An **argument** uses a set of facts or assumptions, called **premises**, to support a **conclusion.**

A **fallacy** is a deceptive argument—an argument in which the conclusion is not well supported by the premises.

BY THE WAY

Advertisements are filled with fallacies, largely because there's usually no really good reason why you should buy some particular brand or product. Still, they must work, because U.S. businesses spend almost $200 billion per year—or nearly $700 per person in the United States—trying to get you to buy stuff.

Common Fallacies

Fallacies in the media are so common that it is nearly impossible to avoid them. Moreover, fallacies often sound persuasive, despite their logical errors, in part because public relations specialists have spent billions of dollars researching how to persuade us to buy products, vote for candidates, or support particular policies. Because fallacies are so common, it is important to be able to recognize them. We therefore begin our study of critical thinking with examples of a few of the most common fallacies. The fallacy in each example has a fancy name, but learning the names is far less important than learning to recognize the faulty reasoning. The experience you gain by analyzing fallacies will provide a foundation upon which to build additional critical thinking skills.

EXAMPLE 1 Appeal to Popularity

"Ford makes the best pickup trucks in the world. More people drive Ford pickups than any other light truck."

Analysis The first step in dealing with any argument is recognizing which statements are premises and which are conclusions. This argument tries to make the case that *Ford makes the best pickup trucks in the world,* so this statement is its conclusion. The only evidence it offers to support this conclusion is the statement *more people drive Ford pickups than any other light truck.* This is the argument's only premise. Overall, this argument has the form

Premise: More people drive Ford pickups than any other light truck.

Conclusion: Ford makes the best pickup trucks in the world.

Note that the original written argument states the conclusion before the premise. Such "backward" structures are common in everyday speech and are perfectly legitimate as long as the argument is well reasoned. In this case, however, the reasoning is faulty.

The fact that more people drive Ford pickups does not necessarily mean that they are the best trucks.

This argument suffers from the fallacy of *appeal to popularity* (or *appeal to majority*), in which the fact that large numbers of people believe or act some way is used inappropriately as evidence that the belief or action is correct. We can represent the general form of this fallacy with a diagram in which the letter *p* stands for a particular statement (Figure 1.1). In this case, *p* stands for the statement *Ford makes the best pickup trucks in the world.*

▶ **Now try Exercise 11.**

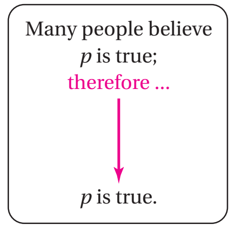


FIGURE 1.1 The fallacy of appeal to popularity. The letters *p* and *q* (used in later diagrams) represent statements.

EXAMPLE 2 False Cause

"I placed the quartz crystal on my forehead, and in five minutes my headache was gone. The crystal made my headache go away."

Analysis We identify the premises and conclusion of this argument as follows:

Premise: I placed the quartz crystal on my forehead.

Premise: Five minutes later my headache was gone.

Conclusion: The crystal made my headache go away.

The premises tell us that one thing (crystal on forehead) happened before another (headache went away), but they don't prove any connection between them. That is, we cannot conclude that the crystal *caused* the headache to go away.

This argument suffers from the fallacy of *false cause*, in which the fact that one event came before another is incorrectly taken as evidence that the first event *caused* the second event. We can represent this fallacy with a diagram in which *A* and *B* represent two different events (Figure 1.2). In this case, *A* is the event of putting the crystal on the forehead and *B* is the event of the headache going away. (We'll discuss how cause *can* be established in Chapter 5.)

▶ **Now try Exercise 12.**

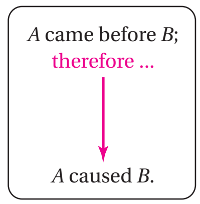


FIGURE 1.2 The fallacy of false cause. The letters *A* and *B* represent events.

EXAMPLE 3 Appeal to Ignorance

"Scientists have not found any concrete evidence of aliens visiting Earth. Therefore, anyone who claims to have seen a UFO must be hallucinating."

Analysis If we strip the argument to its core, it says this:

Premise: There's no proof that aliens have visited Earth.

Conclusion: Aliens have not visited Earth.

The fallacy should be clear: A lack of proof of alien visits does not mean that visits have not occurred. This fallacy is called *appeal to ignorance* because it uses ignorance (lack of knowledge) about the truth of a proposition to conclude the opposite (Figure 1.3). We sometimes sum up this fallacy with the statement "*An absence of evidence is not evidence of absence.*"

▶ **Now try Exercise 13.**

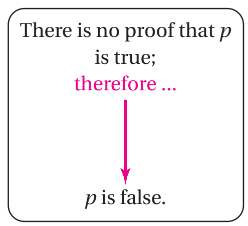


FIGURE 1.3 The fallacy of appeal to ignorance.

Time Out to Think Suppose a person is tried for a crime and found *not* guilty. Can you conclude that the person is innocent? Why or why not? Why do you think our legal system demands that prosecutors prove guilt, rather than demanding that defendants (suspects) prove innocence? How is this idea related to the fallacy of appeal to ignorance?

EXAMPLE 4 Hasty Generalization

"Two cases of childhood leukemia have occurred along the street where the high-voltage power lines run. The power lines must be the cause of these illnesses."

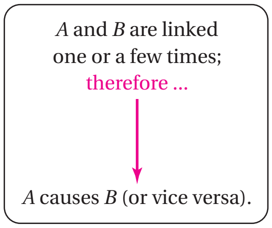


FIGURE 1.4 The fallacy of hasty generalization.

Analysis The premise of this argument cites two cases of leukemia, but two cases are not enough to establish a pattern, let alone to conclude that the power lines caused the illnesses.

The fallacy here is *hasty generalization,* in which a conclusion is drawn from an inadequate number of cases or cases that have not been sufficiently analyzed. If any connection between power lines and leukemia exists, it would have to be established with far more evidence than is provided in this argument. (In fact, decades of research have found no connection between power lines and illness.) We can represent this fallacy with a diagram in which *A* and *B* represent two linked events (Figure 1.4).

▶ **Now try Exercise 14.**

EXAMPLE 5 Limited Choice

"You don't support the President, so you are not a patriotic American."

Analysis This argument has the form

Premise: You don't support the President.
Conclusion: You are not a patriotic American.

The argument suggests that there are only two types of Americans: patriotic ones who support the President and unpatriotic ones who don't. But there are many other possibilities, such as being patriotic while disliking a particular President.

This fallacy is called *limited choice* (or *false dilemma*) because it artificially precludes choices that ought to be considered. Figure 1.5 shows one common form of this fallacy. Limited choice also arises with questions such as "Have you stopped smoking?" Because both *yes* and *no* answers imply that you smoked in the past, the question precludes the possibility that you never smoked. (In legal proceedings, questions of this type are disallowed because they attempt to "lead the witness.") Another simple and common form of this fallacy is "You're wrong, so I must be right." ▶ **Now try Exercise 15.**

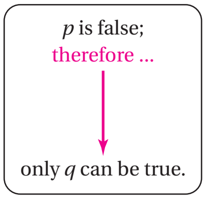


FIGURE 1.5 The fallacy of limited choice.

EXAMPLE 6 Appeal to Emotion

In ads for Michelin tires, a picture of a baby is shown with the words "because so much is riding on your tires."

Analysis If we can consider this an argument at all, it has the form

Premise: You love your baby.
Conclusion: You should buy Michelin tires.

The advertisers hope that the love you feel for a baby will make you want to buy their tires. This attempt to evoke an emotional response as a tool of persuasion represents the fallacy of *appeal to emotion.* Figure 1.6 shows its form when the emotional response is positive. Sometimes the appeal is to negative emotions. For example, the statement *if my opponent is elected, your tax burden will rise* tries to convince you that electing the other candidate will lead to consequences you won't like. (In this negative form, the fallacy is sometimes called *appeal to force.*) ▶ **Now try Exercise 16.**

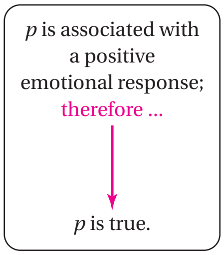


FIGURE 1.6 The fallacy of appeal to emotion.

EXAMPLE 7 Personal Attack

Gwen: You should stop drinking because it's hurting your grades, endangering people when you drink and drive, and destroying your relationship with your family.

Merle: I've seen you drink a few too many on occasion yourself!

Analysis Gwen's argument is well reasoned, with premises offering strong support for her conclusion that Merle should stop drinking. Merle rejects this argument by noting that Gwen sometimes drinks too much herself. Even if Merle's claim is true, it is irrelevant to Gwen's point. Merle has resorted to attacking Gwen personally rather than arguing logically, so we call this fallacy *personal attack* (Figure 1.7). (It is also called *ad hominem,* Latin for "to the person.")

The fallacy of personal attack can also apply to groups. For example, someone might say, "This new bill will be an environmental disaster because its sponsors received large campaign contributions from oil companies." This argument is fallacious because it doesn't challenge the provisions of the bill, but only questions the motives of the sponsors. ▶ **Now try Exercise 17.**

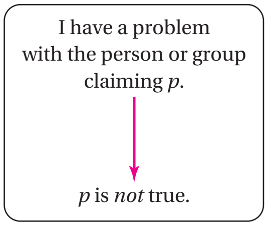


FIGURE 1.7 The fallacy of personal attack.

Time Out to Think A person's (or group's) character, circumstances, and motives occasionally *are* logically relevant to an argument. That is why, for example, witnesses in criminal cases often are asked questions about their personal lives. If you were a judge, how would you decide when to allow such questions?

EXAMPLE 8 Circular Reasoning

"Society has an obligation to provide health insurance because health care is a right of citizenship."

Analysis This argument states the conclusion (*society has an obligation to provide health insurance*) before the premise (*health care is a human right*). But if you read carefully, you'll recognize that the premise and the conclusion both say essentially the same thing, as social obligations are generally based on definitions of accepted rights. This argument therefore suffers from *circular reasoning* (Figure 1.8). ▶ **Now try Exercise 18.**

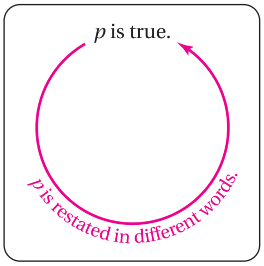


FIGURE 1.8 The fallacy of circular reasoning.

EXAMPLE 9 Diversion (Red Herring)

"We should not continue to fund cloning research because there are so many ethical issues involved. Decisions are based on ethics, and we cannot afford to have too many ethical loose ends."

Analysis The argument begins with its conclusion—*we should not continue to fund cloning research.* However, the discussion is all about ethics. This argument represents the fallacy of *diversion* (Figure 1.9) because it attempts to divert attention from the real issue (funding for cloning research) by focusing on another issue (ethics). The issue to which attention is diverted is sometimes called a *red herring.* (A herring is a fish that turns red when rotten. Use of the term *red herring* to mean a diversion can be traced back to the 19th century, when British fugitives discovered that they could divert bloodhounds from their pursuit by rubbing a red herring across their trail.) Note that personal attacks (see Example 7) are often used as diversions. ▶ **Now try Exercise 19.**

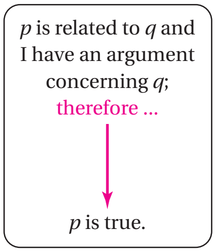


FIGURE 1.9 The fallacy of diversion.

EXAMPLE 10 Straw Man

Suppose that the mayor of a large city proposes decriminalizing drug possession in order to reduce overcrowding in jails and save money on enforcement. His challenger in the upcoming election says, "The mayor doesn't think there's anything wrong with drug use, but I do."

Analysis The mayor did not say that drug use is acceptable. His proposal for decriminalization is designed to solve another problem—overcrowding of jails—and tells us nothing about his general views on drug use. The speaker has distorted the mayor's views. Any argument based on a distortion of someone's words or beliefs is called a *straw man* (Figure 1.10). The term comes from the idea that the speaker has used a